# Transportation Problem

***Introduction***

A special class of linear programming problem is **Transportation Problem**, where the objective is to minimize the cost of distributing a product from a number of **sources** (e.g. factories) to a number of **destinations** (e.g. warehouses) while satisfying both the supply limits and the demand requirement. Because of the special structure of the Transportation Problem the Simplex Method of solving is unsuitable for the Transportation Problem. The model assumes that the distributing cost on a given rout is directly proportional to the number of units distributed on that route. Generally, the transportation model can be extended to areas other than the direct transportation of a commodity, including among others, inventory control, employment scheduling, and personnel assignment.

The transportation problem special feature is illustrated here with the help of following Example

**Example**

Suppose a manufacturing company owns three factories (sources) and distribute his products to five different retail agencies (destinations). The following table shows the capacities of the three factories, the quantity of products required by the various retail agencies and the cost of shipping one unit of the product from each of the three factories to each of the five retail agencies.

| | Retail Agency | | | | | |
|---|---|---|---|---|---|---|
| Factories | 1 | 2 | 3 | 4 | 5 | Capacity |
| 1 | 1 | 9 | 13 | 36 | 51 | 50 |
| 2 | 24 | 12 | 16 | 20 | 1 | 100 |
| 3 | 14 | 33 | 1 | 23 | 26 | 150 |
| Requirement | 100 | 60 | 50 | 50 | 40 | 300 |

Usually the above table is referred as Transportation Table, which provides the basic information regarding the transportation problem. The quantities inside the table are known as transportation cost per unit of product. The capacity of the factories 1, 2, 3 is 50, 100 and 150 respectively. The requirement of the retail agency 1, 2, 3, 4, 5 is 100,60,50,50, and 40 respectively.

In this case, the transportation cost of one unit

from factory 1 to retail agency 1 is 1,

from factory 1 to retail agency 2 is 9,

from factory 1 to retail agency 3 is 13, and so on.

A transportation problem can be formulated as linear programming problem using variables with two subscripts.

Let

$x_{11}$=Amount to be transported from factory 1 to retail agency 1

$x_{12}$= Amount to be transported from factory 1 to retail agency 2

……..

……..

……..

……..

$x_{35}$= Amount to be transported from factory 3 to retail agency 5.

Let the transportation cost per unit be represented by $C_{11}$, $C_{12}$, …..$C_{35}$ that is $C_{11}=1$, $C_{12}=9$, and so on.

Let the capacities of the three factories be represented by $a_1=50$, $a_2=100$, $a_3=150$.

Let the requirement of the retail agencies are $b_1=100$, $b_2=60$, $b_3=50$, $b_4=50$, and $b_5=40$. Thus, the problem can be formulated as

Minimize

$$C_{11}x_{11}+C_{12}x_{12}+\ldots\ldots\ldots\ldots\ldots+C_{35}x_{35}$$

Subject to:

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = a1$$
$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = a2$$
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = a3$$

$$x11 + x21 + x31 = b1$$
$$x_{12} + x_{22} + x_{32} = b_2$$
$$x_{13} + x_{23} + x_{33} = b_3$$
$$x14 + x24 + x34 = b4$$
$$x15 + x25 + x35 = b5$$

$$x_{11}, x_{12}, \ldots\ldots, x_{35} \geq 0.$$

Thus, the problem has 8 constraints and 15 variables. So, it is not possible to solve such a problem using simplex method. This is the reason for the need of special computational procedure to solve transportation problem. There are varieties of procedures, which are described in the next section.

## Transportation Algorithm

The steps of the transportation algorithm are exact parallels of the simplex algorithm, they are:

**Step 1:** Determine a starting basic feasible solution, using any one of the following three methods

North West Corner Method
Least Cost Method
Vogel Approximation Method

**Step 2:** Determine the optimal solution using the following
method 1
MODI (Modified Distribution Method) or UV
Method.

## Basic Feasible Solution of a Transportation Problem

The computation of an initial feasible solution is illustrated in this section with the help of the Example discussed in the previous section. The problem in the example has 8 constraints and 15 variables we can eliminate one of the constraints since $a_1 + a_2 + a_3 = b_1 + b_2 + b_3 + b_4 + b_5$. Thus now the problem contains 7 constraints and 15 variables. Note that any initial (basic) feasible solution has at most 7 non-zero $X_{ij}$. Generally, any basic feasible solution with m sources (such as factories) and n destination (such as retail agency) has at most m + n -1 non-zero $X_{ij}$.
The special structure of the transportation problem allows securing a non artificial basic feasible solution using one the following three methods.

North West Corner Method
Least Cost Method
Vogel Approximation Method

The difference among these three methods is the **quality** of the initial basic feasible solution they produce, in the sense that a better that a better initial solution yields a smaller objective value. Generally the Vogel Approximation Method produces the **best** initial basic feasible solution, and the North West Corner Method produces the **worst**, but the North West Corner Method involves least computations.

**North West Corner Method**:

The method starts at the North West (upper left) corner cell of the tableau (variable $x_{11}$).

**Step -1:** Allocate as much as possible to the selected cell, and adjust the associated amounts of capacity (supply) and requirement (demand) by subtracting the allocated amount.

**Step -2:** Cross out the row (column) with zero supply or demand to indicate that no further assignments can be made in that row (column). If both the row and column becomes zero simultaneously, cross out one of them only, and leave a zero supply or demand in the uncrossed out row (column).

**Step -3:** If exactly one row (column) is left uncrossed out, then stop. Otherwise, move to the cell to the right if a column has just been crossed or the one below if a row has been crossed out. Go to step -1.

Consider the problem discussed in the above Example to illustrate the North West Corner Method of determining basic feasible solution.

| | Retail Agency | | | | | |
|---|---|---|---|---|---|---|
| Factories | 1 | 2 | 3 | 4 | 5 | Capacity |
| 1 | 1 | 9 | 13 | 36 | 51 | 50 |
| 2 | 24 | 12 | 16 | 20 | 1 | 100 |
| 3 | 14 | 33 | 1 | 23 | 26 | 150 |
| Requirement | 100 | 60 | 50 | 50 | 40 | 300 |

The allocation is shown in the following tableau:

| | | | | | | **Capacity** |
|---|---|---|---|---|---|---|
| | 1<br>**50** | 9 | 13 | 36 | 51 | ~~**50**~~ |
| | 24<br>**50** | 12<br>**50** | 16 | 20 | 1 | ~~**100**~~ ~~**50**~~ |
| | 14 | 33<br>**10** | 1<br>**50** | 23<br>**50** | 26<br>**40** | ~~**150**~~ ~~**140**~~ ~~**90**~~ ~~**40**~~ |
| **Requirement** | ~~**100**~~<br>**50** | ~~**60**~~<br>~~**10**~~ | ~~**50**~~ | ~~**50**~~ | ~~**40**~~ | |

The arrows show the order in which the allocated (**bolded**) amounts are generated. The starting basic solution is given as

$x_{11} = 50,$

$x_{21} = 50, x_{22} = 50$

$x_{32} = 10, x_{33} = 50, x_{34} = 50, x_{35} = 40$

The corresponding transportation cost is

50*1+50*24+50*12+10*33+50*1+50*23+40*26=4420

It is clear that as soon as a value of $X_{ij}$ is determined, a row (column) is eliminated from further consideration. The last value of $X_{ij}$ eliminates both a row and column. Hence a feasible solution computed by North West Corner Method can have at most m + n – 1 positive $X_{ij}$ if the transportation problem has m sources and n destinations.

**Least Cost Method**

The least cost method is also known as matrix minimum method in the sense we look for the row and the column corresponding to which $C_{ij}$ is minimum. This method finds a better initial basic feasible solution by concentrating on the cheapest routes. Instead of starting the allocation with the northwest cell as in the North West Corner Method, we start by allocating as much as possible to the cell with the smallest unit cost. If there are two or more minimum costs then we should select the row and the column corresponding to the lower numbered row. If they appear in the same row we should select the lower numbered column. We then cross out the satisfied row or column, and adjust the amounts of capacity and requirement accordingly. If both a row and a column is satisfied simultaneously, only one is crossed out. Next, we look for the uncrossed-out cell with the smallest unit cost and repeat the process until we are left at the end with exactly one uncrossed-out row or column.

The least cost method of determining initial basic feasible solution is illustrated with the help of problem presented above

| | | | | | | **Capacity** |
|---|---|---|---|---|---|---|
| | 1<br>**50** | 9 | 13 | 36 | 51 | **~~50~~** |
| | 24 | 12<br>**60** | 16 | 20 | 1<br>**40** | **~~100~~ 60** |
| | 14<br>**50** | 33 | 1<br>**50** | 23<br>**50** | 26 | **~~150~~ ~~100~~ ~~50~~** |
| **Requirement** | **~~100~~ ~~50~~** | **~~60~~** | **~~50~~** | **~~50~~** | **~~40~~** | |

The Least Cost method is applied in the following manner:

We observe that $C_{11}$=1 is the minimum unit cost in the table. Hence $X_{11}$=50 and the first row is crossed out since the row has no more capacity. Then the minimum unit cost in the uncrossed-out row and column is $C_{25}$=1, hence $X_{25}$=40 and the fifth column is crossed out. Next $C_{33}$=1is the minimum unit cost, hence $X_{33}$=50 and the third column is crossed out. Next $C_{22}$=12 is the minimum unit cost, hence $X_{22}$=60 and the second column is crossed out. Next we look for the uncrossed-out row and column now $C_{31}$=14 is the minimum unit cost, hence $X_{31}$=50 and crossed out the first column since it was satisfied. Finally $C_{34}$=23 is the minimum unit cost, hence $X_{34}$=50 and the fourth column is crossed out.

So that the basic feasible solution developed by the Least Cost Method has transportation cost is

$$1*50+12*60+1*40+14*50+1*50+23*50=2710$$

**Note** that the minimum transportation cost obtained by the least cost method is much lower than the corresponding cost of the solution developed by using the north-west corner method.